

Explaining Missing Pennies in Amortization Schedules

We will be using the information given in: [13.3 AMORTIZATION SCHEDULE](#) from [Business Math: A Step-by-Step Handbook Abridged](#) by Sanja Krajisnik, Carol Leppinen and Jelena Loncar-Vines. Anything written here in purple is copied from that text.

The Handbook expects the user to have access to a calculator, the BAII Plus (from Texas Instruments). We will work around this by using Excel for our calculations.

From the text:

Example 13.3.1: Payment Plan on a Dishwasher

Tamara purchased a new dishwasher from The Bay for **\$895.94**. By placing it on her Bay credit card, she can pay off the dishwasher through a special **six-month** payment plan promotion that charges her **5.9% compounded monthly**. Construct the complete amortization schedule for Tamara and total her interest charges.

Solution:

Construct a complete amortization schedule for the dishwasher payments along with the total interest paid.

Payment Number	Payment Amount (\$) (PMT)	Interest Portion (\$) (INT)	Principal Portion (\$) (PRN)	Principal Balance Remaining (\$) (BAL)
0-Start				\$895.94
1	\$151.90	\$4.41	\$147.49	\$748.45
2	\$151.90	\$3.68	\$148.22	\$600.22
3	\$151.90	\$2.95	\$148.95	\$451.28
4	\$151.90	\$2.22	\$149.68	\$301.59
5	\$151.90	\$1.48	\$150.42	\$151.18
6	\$151.90 + BAL	\$0.74	\$151.16 + BAL	\$0.02
Total	\$911.42	\$15.48	\$895.94 + BAL	

Table 13.3.2. Payment Schedule for Example 13.3.1

Payment Number	Payment Amount (\$) (PMT)	Interest Portion (\$) (INT)	Principal Portion (\$) (PRN)	Principal Balance Remaining (\$) (BAL)
0-Start				\$895.94
1	\$151.90	\$4.41	\$147.49	\$748.45
2	\$151.90	\$3.67	\$148.23	\$600.22
3	\$151.90	\$2.96	\$148.94	\$451.28
4	\$151.90	\$2.21	\$149.69	\$301.59
5	\$151.90	\$1.49	\$150.41	\$151.18
6	\$151.92	\$0.74	\$151.18	\$0.00
Total	\$911.42	\$15.48	\$895.94	

Table 13.3.3. Amortization Schedule for Example 13.3.1

Why are there two tables? The first, Table 13.3.2, is an interim step in the calculations, used to calculate the final payment amount; the second, Table 13.3.3, is the final schedule, showing corrections for 'missed pennies' in bold (months 2-5).

In more detail, we will show how these figures are arrived at.

The monthly payment amount

Let's start with the calculation of the payment amount (PMT in the table above); in Excel, we can calculate:

`=PMT(0.059/12, 6, 895.94, 0, 0)`

which is **-£151.903441** to 6 dp (Excel gives the value as negative).

This figure is the most important, as it drives all further calculations.

Recreating Table 13.3.3

In three steps, we can create the final amortization schedule shown in the text.

Part 1 : No rounding

First, we'll do the calculations in Excel, with no rounding at all (shown to 6dp).

Month	Start BAL	INT	PMT	PRN	End BAL
1	£895.94	£4.405038	£151.903441	£147.498403	£748.441597
2	£748.441597	£3.679838	£151.903441	£148.223603	£600.217994
3	£600.217994	£2.951072	£151.903441	£148.952370	£451.265624
4	£451.265624	£2.218723	£151.903441	£149.684719	£301.580905
5	£301.580905	£1.482773	£151.903441	£150.420669	£151.160237
6	£151.160237	£0.743204	£151.903441	£151.160237	£0.000000
			£911.420648	£895.940000	

where 'End BAL' = 'Start BAL' minus PRN, and INT is calculated from the interest rate and the 'Start BAL'. (Note that we've used start/end balance columns, instead of having month 0.)

As expected, the total principal paid is equal to the original loan amount, and the final balance is zero. (Actually, in Excel the final balance is $-2.47e-12$ and there is a difference of $-2.50e-12$ between the total principal and the loan amount, due to working with floating point numbers; for our purposes, we can pretend they're zero.)

Clearly, we can't deal in fractions of pennies, so we begin the process of rounding until all (a) all the figures are 2dp; and (b) our figures are numerically as close to the un-rounded values as possible.

The total repayment amount that we're going to ask the customer to repay is $£151.903441 \times 6 = £911.420648$, which we round down* to **£911.42**. We will call this figure **immutable** : nothing we do from here on should change this value, **£911.42** is the actual full amount that we will require to be repaid. Trivially, the total interest charged will be $£911.42 - £895.94 = £15.48$. This figure is clearly also immutable.

Unless this total amount (as pence) is divisible by the number of payments (n), the monthly payments won't be equal. It is standard to have the first (n-1) payments be equal, and the nth payment can be a few pence either way.

Hence for our example, $£911.42 / 6 = £151.903333$ which we round (up or down) to **£151.90**. $£911.42 - (5 \times £151.90) = £151.92$. The first 5 payments will be **£151.90** and the final payment will be **£151.92**.

(*) When rounding final totals (outside of internal calculations), it is common to round figures in the customer's favour; in this case, we round down.

Part 2: Using actual payment amounts, then rounding

The schedule is calculated the same as above, except we introduce the actual payment amounts:

Month	Start BAL	INT	PMT	PRN	End BAL
1	£895.94	£4.405038	£151.90	£147.494962	£748.445038
2	£748.445038	£3.679855	£151.90	£148.220145	£600.224893
3	£600.224893	£2.951106	£151.90	£148.948894	£451.275999
4	£451.275999	£2.218774	£151.90	£149.681226	£301.594772
5	£301.594772	£1.482841	£151.90	£150.417159	£151.177613
6	£151.177613	£0.743290	£151.92	£151.176710	£0.000903
			£911.42	£895.939097	

Of course, now the final balance and total principal values are incorrect, because the payments don't cover the exact interest amounts.

Next, we round all the above figures to 2dp:

Month	Start BAL	INT	PMT	PRN	End BAL
1	£895.94	£4.410000	£151.90	£147.490000	£748.450000
2	£748.450000	£3.680000	£151.90	£148.220000	£600.220000
3	£600.220000	£2.950000	£151.90	£148.950000	£451.280000
4	£451.280000	£2.220000	£151.90	£149.680000	£301.590000
5	£301.590000	£1.480000	£151.90	£150.420000	£151.180000
6	£151.180000	£0.740000	£151.92	£151.180000	£0.000000
			£911.42	£895.940000	

This is essentially Table 13.3.2.

The final balance and total principal values are now correct. But what about the monthly calculations?

Part 3: Missing pennies

Look at month 2: the end balance should be **£748.45 - £148.22 = £600.23**, not **£600.22**, it is a penny out. This is also true of months 3-5, which are also a penny out.

From the original text:

The "Missing Penny"

In the creation of the amortization schedule, you always round the numbers off to two decimals since you are dealing with currency. However, as per the rules of rounding, you do not round any numbers in your calculations until you reach the end of the amortization schedule and the annuity has been reduced to zero.

As a result, you have a triple rounding situation involving the balance along with the principal and interest components on every line of the table. What sometimes happens is that a "missing penny" occurs and the schedule needs to be corrected as per step 12 of the process above. In other words, calculations will sometimes appear to be off by a penny. You can identify the "missing penny" when one of the two standard calculations using the rounded numbers from the schedule becomes off by a penny.

In these instances of the "missing penny", you adjust the schedule as needed to ensure that the math works properly at all times. The golden rule, though, is that the balance in the account (BAL) is always correct and should NEVER be adjusted. Follow this order in making any adjustments:

Adjust the PRN if necessary such that the previous BAL – PRN = current BAL.

Then adjust INT if necessary such that PMT – PRN = INT

Usually these adjustments come in pairs, meaning that if you need to adjust the PRN up by a penny, somewhere later in the schedule you will need to adjust the PRN down by a penny. Ultimately, these changes in most circumstances have no impact on the total interest (INT) or total principal (PRN) components, since the "missing penny" is nothing more than a rounding error within the schedule.

Note that from now on, we treat the start/end balances as **immutable**: from the above, "The golden rule, though, is that the balance in the account (BAL) is always correct and should NEVER be adjusted." We can alter the INT and PRN amounts to make corrections, but nothing else.

Let's correct month 2:

PRN for month 2 is shown as **£148.22**. As the end balance £600.22 is fixed, then the new PRN is the difference between the month 1 and month 2 balances, **£748.45 - £600.22 = £148.23**. Additionally, the INT for month 2 is **£3.68**. But our payment minus our new PRN is **£151.90 - £148.23 = £3.67**, which is our new INT figure.

If we repeat this for months 3-5, we get the following schedule:

Month	Start BAL	INT	PMT	PRN	End BAL
1	£895.94	£4.41	£151.90	£147.49	£748.45
2	£748.45	£3.67	£151.90	£148.23	£600.22
3	£600.22	£2.96	£151.90	£148.94	£451.28
4	£451.28	£2.21	£151.90	£149.69	£301.59
5	£301.59	£1.49	£151.90	£150.41	£151.18
6	£151.18	£0.74	£151.92	£151.18	£0.00
		£15.48	£911.42	£895.94	

which agrees with Table 13.3.3 from the text. Thus we have accounted for the ‘missing pennies’.

Final notes

We don't really need to identify the missing pennies

In reality, we don't need to identify the ‘missing pennies’; we may as well just calculate INT and PRN for every month once we have the (immutable) balances. The balances determine the principal (PRN), and the principal and the payments determine the interest amount (INT), so we could do those calculations as a matter of course for all months.

How do we know INT and PRN sum correctly?

How do we know that, post-correction, the sums of INT and PRN will add up to their fixed totals?

Let L_0 be the initial balance (original loan amount), and L_n be the balance at the end of month n ; let p be the monthly payment for months 1 to $(n-1)$, and p_n be the final monthly payment.

If we do the ‘missing penny’ corrections, using the variables defined above, we get:

Month	Start BAL	INT	PMT	PRN	End BAL
1	L_0	$=(p-(L_0-L_1))$	p	$=(L_0-L_1)$	L_1
2	L_1	$=(p-(L_1-L_2))$	p	$=(L_1-L_2)$	L_2
3	L_2	$=(p-(L_2-L_3))$	p	$=(L_2-L_3)$	L_3
4	L_3	$=(p-(L_3-L_4))$	p	$=(L_3-L_4)$	L_4
5	L_4	$=(p-(L_4-L_5))$	p	$=(L_4-L_5)$	L_5
6	L_5	$=p_n - L_5$	p_n	$=(L_5-L_6)=L_5$	$L_6 = 0.00$

Note that the INT column sums to $((5*p)+p_n) - L_0$, i.e. the total payment minus the original loan amount, which is exactly the total interest charged. Similarly, the PRN column sums to L_0 . The calculations done this way will always tally correctly.